FORMULAIRE : FONCTIONS HYPERBOLIQUES

1. Relations

– $\forall x \in \mathbb{R}$ $\quad \mathrm{ch}(x) = \dfrac{e^x + e^{-x}}{2}$; $\mathrm{sh}(x) = \dfrac{e^x - e^{-x}}{2}$

– $\forall x \in \mathbb{R}$ $\quad \mathrm{th}(x) = \dfrac{\mathrm{sh}(x)}{\mathrm{ch}(x)} = \dfrac{e^x - e^{-x}}{e^x + e^{-x}} = \dfrac{e^{2x} - 1}{e^{2x} + 1} = \dfrac{1 - e^{-2x}}{1 + e^{-2x}}$

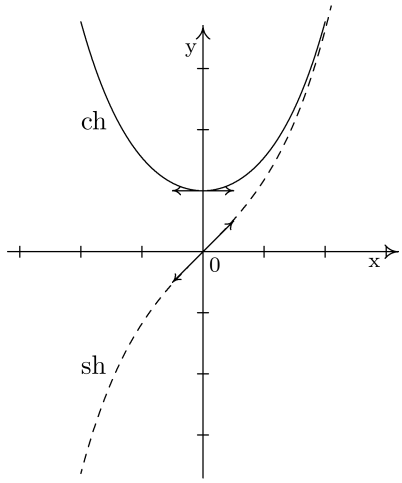
– $\forall x \in \mathbb{R}^*$ $\quad \coth(x) = \dfrac{\mathrm{ch}(x)}{\mathrm{sh}(x)} = \dfrac{e^x + e^{-x}}{e^x - e^{-x}} = \dfrac{e^{2x} + 1}{e^{2x} - 1} = \dfrac{1 + e^{-2x}}{1 - e^{-2x}}$

– $\forall x \in \mathbb{R}$ $\quad \mathrm{ch}^2(x) - \mathrm{sh}^2(x) = 1$; $\mathrm{ch}(x) + \mathrm{sh}(x) = e^x$; $\mathrm{ch}(x) - \mathrm{sh}(x) = e^{-x}$

– $\forall x \in \mathbb{R}$ $\quad 1 - \mathrm{th}^2(x) = \dfrac{1}{\mathrm{ch}^2(x)}$; $\quad \forall x \in \mathbb{R}^*$ $\quad 1 - \coth^2(x) = \dfrac{-1}{\mathrm{sh}^2(x)}$

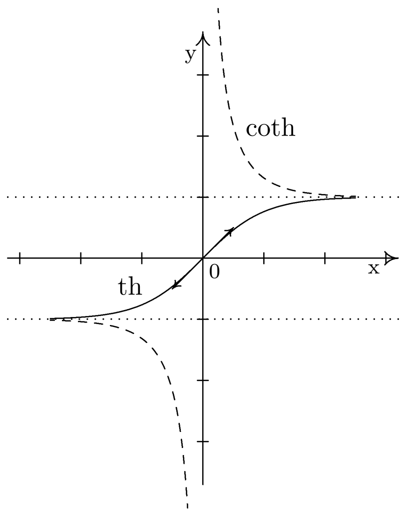
2. Variations et représentations graphiques

$$\forall x \in \mathbb{R} \quad \begin{cases} \mathrm{ch}(-x) = \mathrm{ch}(x) \quad ; \quad \mathrm{sh}(-x) = -\mathrm{sh}(x) \\ (\mathrm{ch})'(x) = \mathrm{sh}(x) \quad ; \quad (\mathrm{sh})'(x) = \mathrm{ch}(x) \\ \mathrm{ch}(x) \geqslant 1 \end{cases}$$



$$\forall x \in \mathbb{R} \quad \begin{cases} \mathrm{th}(-x) = -\mathrm{th}(x) \\ (\mathrm{th})'(x) = 1 - \mathrm{th}^2(x) \\ -1 < \mathrm{th}(x) < 1 \end{cases}$$

$$\forall x \in \mathbb{R}^* \begin{cases} \coth(-x) = -\coth(x) \\ (\coth)'(x) = 1 - \coth^2(x) \\ |\coth(x)| > 1 \end{cases}$$



3. Formules d'addition

$\mathrm{ch}(a+b) = \mathrm{ch}(a)\mathrm{ch}(b) + \mathrm{sh}(a)\mathrm{sh}(b) \qquad \mathrm{ch}(a-b) = \mathrm{ch}(a)\mathrm{ch}(b) - \mathrm{sh}(a)\mathrm{sh}(b)$

$\mathrm{sh}(a+b) = \mathrm{sh}(a)\mathrm{ch}(b) + \mathrm{sh}(b)\mathrm{ch}(a) \qquad \mathrm{sh}(a-b) = \mathrm{sh}(a)\mathrm{ch}(b) - \mathrm{sh}(b)\mathrm{ch}(a)$

$\mathrm{th}(a+b) = \dfrac{\mathrm{th}(a) + \mathrm{th}(b)}{1 + \mathrm{th}(a)\mathrm{th}(b)} \qquad \mathrm{th}(a-b) = \dfrac{\mathrm{th}(a) - \mathrm{th}(b)}{1 - \mathrm{th}(a)\mathrm{th}(b)}$

4. Formules de duplication

- $\mathrm{ch}(2a) = \mathrm{ch}^2(a) + \mathrm{sh}^2(a) = 2\mathrm{ch}^2(a) - 1 = 1 + 2\mathrm{sh}^2(a)$

$$\mathrm{ch}^2(a) = \frac{\mathrm{ch}(2a)+1}{2} \qquad \mathrm{sh}^2(a) = \frac{\mathrm{ch}(2a)-1}{2}$$
$$\mathrm{sh}(2a) = 2\mathrm{sh}(a)\mathrm{ch}(a) \qquad \mathrm{th}(2a) = \frac{2\mathrm{th}(a)}{1+\mathrm{th}^2(a)}$$

- En posant $t = \mathrm{th}\left(\frac{a}{2}\right)$:

$$\mathrm{ch}(a) = \frac{1+t^2}{1-t^2} \qquad \mathrm{sh}(a) = \frac{2t}{1-t^2} \qquad \mathrm{th}(a) = \frac{2t}{1+t^2} \qquad \mathrm{e}^a = \frac{1+t}{1-t}$$

- $\mathrm{ch}(3a) = 4\mathrm{ch}^3(a) - 3\mathrm{ch}(a) \qquad \mathrm{sh}(3a) = 4\mathrm{sh}^3(a) + 3\mathrm{sh}(a)$

$$\mathrm{th}(3a) = \frac{3\mathrm{th}(a) + \mathrm{th}^3(a)}{1 + 3\mathrm{th}^2(a)}$$

5. Transformation de produits en sommes, de sommes en produits

- $\mathrm{ch}(a)\mathrm{ch}(b) = \frac{1}{2}\big[\mathrm{ch}(a+b) + \mathrm{ch}(a-b)\big] \quad ; \quad \mathrm{sh}(a)\mathrm{sh}(b) = \frac{1}{2}\big[\mathrm{ch}(a+b) - \mathrm{ch}(a-b)\big]$

$$\mathrm{sh}(a)\mathrm{ch}(b) = \frac{1}{2}\big[\mathrm{sh}(a+b) + \mathrm{sh}(a-b)\big]$$

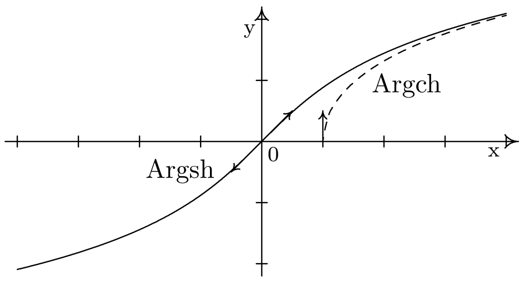
- $\mathrm{ch}(p) + \mathrm{ch}(q) = 2\mathrm{ch}\left(\frac{p+q}{2}\right)\mathrm{ch}\left(\frac{p-q}{2}\right)$

$$\mathrm{ch}(p) - \mathrm{ch}(q) = 2\mathrm{sh}\left(\frac{p+q}{2}\right)\mathrm{sh}\left(\frac{p-q}{2}\right)$$
$$\mathrm{sh}(p) + \mathrm{sh}(q) = 2\mathrm{sh}\left(\frac{p+q}{2}\right)\mathrm{ch}\left(\frac{p-q}{2}\right)$$
$$\mathrm{sh}(p) - \mathrm{sh}(q) = 2\mathrm{sh}\left(\frac{p-q}{2}\right)\mathrm{ch}\left(\frac{p+q}{2}\right)$$
$$\mathrm{th}(p) + \mathrm{th}(q) = \frac{\mathrm{sh}(p+q)}{\mathrm{ch}(p)\mathrm{ch}(q)} \quad ; \quad \mathrm{th}(p) - \mathrm{th}(q) = \frac{\mathrm{sh}(p-q)}{\mathrm{ch}(p)\mathrm{ch}(q)}$$

6. Fonctions hyperboliques réciproques

$$\forall x \geqslant 1 \quad \begin{cases} \mathrm{Argch}(x) = \ln(x + \sqrt{x^2-1}) \\ \mathrm{Argch}'(x) = \dfrac{1}{\sqrt{x^2-1}} \end{cases}$$

$$\forall x \in \mathbb{R} \quad \begin{cases} \mathrm{Argsh}(x) = \ln(x + \sqrt{x^2+1}) \\ \mathrm{Argsh}'(x) = \dfrac{1}{\sqrt{x^2+1}} \end{cases}$$



$$\forall x \in]-1,1[\quad \begin{cases} \mathrm{Argth}(x) = \dfrac{1}{2}\ln\left(\dfrac{1+x}{1-x}\right) \\ \mathrm{Argth}'(x) = \dfrac{1}{1-x^2} \end{cases}$$

$$\forall x \in \mathbb{R},\ |x| > 1 \quad \begin{cases} \mathrm{Argcoth}(x) = \dfrac{1}{2}\ln\left(\dfrac{x+1}{x-1}\right) \\ \mathrm{Argcoth}'(x) = \dfrac{1}{x^2-1} \end{cases}$$